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LINEAR PROGRAMMING
IN THE INDONESIAN AIR FORCE'S
MAINTENANCE OPERATIONS

by

Sumardjo

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LINEAR PROGRAMMING IN THE INDONESIAN AIR FORCE'S MAINTENANCE OPERATIONS

by

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ABSTRACT

This study demonstrates the feasibility of using linear programming to determine the allocation of avionics maintenance needed by the Indonesian Air Force between organizational maintenance facilities and outside contracting. Although the research focusses on avionics maintenance, the general principles are applicable to other military and civilian organizations. This study demonstrates how the application of linear programming techniques can help the Air Force with its strategic planning for maintenance of aircraft. However, it is not currently possible to implement such a system without development of a supporting data collection system. Therefore, it is recommended that development of such a system begin as soon as possible. A logical starting place is within the avionics maintenance organization.

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I. INTRODUCTION

A. BACKGROUND AND PURPOSE OF THESIS

1. National Objectives and Policy Direction

The Republic of Indonesia is a sovereign and democratic state whose development objective is to create a unified and prosperous society which balances material and spiritual welfare in an equitable distribution among Indonesia's population, based upon the state philosophy Pancasila. The allocation of resources and the results of trying to implement national development are synonymous with an earnest attempt to put Pancasila into practice. Development efforts are carried out in a continuous, comprehensive, and integrated manner in stages according to Five-Year Development Plans (Repelita), and within the general framework of Long-Term Development Guidelines sanctioned by the People's Consultative Assembly.

The guidelines of state policy issued by The Consultative Assembly emphasize economic development. Development in other sectors is carried out in conformity with, and in accordance with, the progress achieved in the economic sector. The implementation of this development policy depends upon the Trilogy of Development, which emphasizes maintaining equity in the development process in order to achieve social justice for all, with sufficiently high economic growth, and with sound, dynamic, and stable national policy. The three elements of the Trilogy of Development are interrelated and need to be implemented in a harmonious, integrated and mutually supporting manner.

Twenty five years of national effort have been directed towards implementing the development policy based on these guidelines. The efforts to solve basic problems of development have resulted in many concrete and visible achievements. However, at the same time the dynamics of development have brought about new problems demanding solutions. In addition, all organizations at all levels have to be involved in finding more effective and efficient solutions to achieve the goal of their organization in parallel with the national objective and policy direction [Ref. 1].

2. National Science and Technology Development

In national development, science and technology play a crucial role as industrialization accelerates. Therefore, science and technology have received increasing attention. Similar to with national development, science and technological development is carried out through five main national research and technological programs: primary human needs; energy and natural resources; industrialization; **defense and security;** and society, economy, culture, philosophy, law and legislation. The National Research Council was founded in 1984 and coordinates, formulates, monitors, and evaluates these programs.

The developing industry in Indonesia reflects an increasing capacity to create new technologies, as well as integrate existing ones. In the aviation industry, 214 aircraft have been delivered to customers: 91 NC 212-type planes, 24 CN235-type planes, 19 NBEL412-type helicopters, 84 NBO105-type helicopters, and 10 NAS 332-type helicopters. Fifteen of these were exported. A contract has been signed for the manufacture spare parts for the F-16 jet-fighter, and the Boeing 737, Boeing 767 and F-100 commercial airplanes. Industrial progress also gives challenging tasks to decision makers in maintenance organizations with scarce resources.

3. Armed Forces and Air Force

Currently, Indonesia is one of the fastest developing countries in Southeast Asia. Indonesian's operations and management decisions involve trying to make the most effective use of its resources. Limited resource allocation is a major problem for most organizations in Indonesia, such as the Armed Forces and the Indonesian Air Force, in particular. Indonesian Air Force's resources typically include machinery, personnel, money, time, maintenance facilities and warehouse space, aircraft, avionics equipment, radar, and spare parts.

The Indonesian Armed Forces have a dual function; they are a defense and security force and a social force. The Indonesian Air Force is an integral part of the Armed Forces and it has the following missions: to protect air sovereignty and maintain air superiority above Indonesia's territory, and to develop the national aviation potential. Thus, the Air Force not only has a military mission, it also has a civilian mission as well.

The Air Force of the Republic of Indonesia was established as a separate service in 1946, and evolved from the aviation division of the People's Security Force (BKR). Initially, the Air Force was fairly small and flew mostly United States and Western European aircraft. However, from the early 1950's to the early 1980's, the force expanded rapidly, with a variety of aircraft and avionics equipment [Ref. 2, p. 325]. During the early years the Air Force did not have maintenance facilities for intermediate or depot maintenance capability, and only a few squadrons had flight-line maintenance or organization maintenance facilities with a functional test capability.

> Maintenance is all actions necessary for retaining an item in or restoring it to, a serviceable condition. Maintenance includes all of the following procedures in regards to aircraft: service and repair, modification, modernization, overhaul, inspection, and condition determination [Ref. 3].

Having maintenance facilities for the Indonesian Air Force has some advantages: maintenance costs are cheaper than maintenance contracts for intermediate or long-term, because the Air Force does not pay direct labor costs, taxes and so forth; turn around time is much less and allows for increased availability. The motto of the Air Force or in any military organization is "high availability." The investment in highly-skilled maintenance personnel to improve productivity in the Air Force, Armed Forces, and Government and the transplant of technology will contribute to Indonesian's future development.

By the late 1978's a meeting (a small conference) was held at Air Force Headquarters to evaluate the need to set-up avionics maintenance facilities. The meeting was composed of three officers: Lieutenant Colonel Djati (now Col. Ret.), Captain Sardjono (Air Vice Marshall), and Captain Hadi Prawoto (Colonel). Out of this meeting came an avionics maintenance organization concept for the Air Force. The major reason for establishing avionics maintenance was that during its thirty-eight years existence the Air Force had spent a huge amount of money for maintenance contracts on avionics equipment. Yet many aircraft could not be flown because of avionics problems. The objectives of the avionics maintenance program were to:

- Reduce Maintenance Cost, and
- Create the Ability to Respond Rapidly to Avionics Problems.

To achieve these objectives the Indonesian Air Force needed to invest in training highly skilled personnel and in acquiring advanced technology.

In 1983, the Avionics Maintenance Shops were inaugurated by Air Chief of Staff, and named Skavionics 01 (in Iswayudi AFB, Madiun) and Skavionics 02 (in Halim AFB, Jakarta). The maintenance levels of the Avionics Maintenance Organization are intermediate level and limited depot level. They have the missions to repair and maintain the communication, navigation, airborne radar, guidance system, and other special equipment.

During the past ten years of squadron avionics maintenance, the Air Force has accumulated a great deal of experience in repairing avionics equipment. In this thesis, I will show how the Avionics Maintenance Organization impacts the readiness of aircraft in any air operation and how the Avionics Maintenance Organization could maximize the readiness of aircraft by implementing linear programming in the planning for the repair of avionics equipment. In particular, through linear programming the Avionics Maintenance Organization could reduce the overall maintenance costs while improving the response time.

Readiness is a concept that integrates the diverse factors that affect the ability to deploy, engage, and sustain an effective combat force [Ref. 4]. In other words, readiness is a measure of the ability of military forces, units, weapon systems, equipment, and personnel to perform functions for which they have been designed, organized or trained [Ref. 5]. So, in this case, I define readiness as the percentage of time an aircraft is available for service. For example, there are 24 hours per day, and the total hours per month are 24 x 30 = 720 hours. If an aircraft is available for service for 600 hours during the month, then its readiness is 600/720 or 83 percent.

Linear programming is a management technique that can be used to enhance readiness. This technique employs a mathematical model in which the constraint structure consists of linear systems of equations or inequalities and the objective function is also linear. The linear programming model can be an effective and efficient tool for decision makers to improve the allocation of scarce resources, and can indicate how resources promote readiness. The model can indicate what resources constraints are significant and how those constraints interact in the maintenance operations of the Air Force.

B. RESEARCH OBJECTIVES

The primary research objective is to demonstrate that linear programming can be used in resource allocation decisions in the maintenance operations of the Indonesian Air Force. In particular, linear programming can optimize the use of limited resources to maximum readiness in the Air Force. The secondary objectives are to identify the significant factors impacting readiness, and to demonstrate how linear programming can assist decision makers in focussing on these factors in the decision making process. Finally, this study will show what information is required by decision makers to use linear programming to support the decision making process in the Indonesian Air Force maintenance facilities.

C. THESIS SCOPE AND LIMITATIONS

The thesis surveys the current literature and presents some examples of military and relevant non-military uses of linear programming. This thesis then develops a linear programming application as an example of its possible uses in the Indonesian Air Force. The outputs from that example are discussed in detail.

D. METHODOLOGY

The methodology employed in this thesis research effort is to collect information relevant to linear programming by researching reference books and periodicals dealing with production and operation management systems and linear programming models. Then, as an example, the author will formulate and solve a specific problem in the Indonesian Air Force. The author will use a mixture of avionics production and maintenance data from author's experience during the last ten years, to simulate the maintenance activities of Indonesian Air Force after ten or fifteen years of operation.

E. ORGANIZATION OF THE THESIS

The thesis is divided into four chapters. Chapter I is the introduction which covers the background and purpose of the thesis. Chapter II discusses linear programming and formulation from a theoretical perspective, and reviews the results from relevant applications. Chapter III is a formulation and discussion of an example problem associated with the Indonesian Air Force Avionics Maintenance Organization. Chapter IV contains a summary, presents conclusions, and makes recommendations for using linear programming.

II. LINEAR PROGRAMMING AND MODEL FORMULATION

A. WHAT IS LINEAR PROGRAMMING?

> Linear programming[1] [Ref. 6] is a technique for specifying how to use limited resources or capacities of business to obtain a particular objective, such as least cost, highest margin or least time, when those resources have alternative uses. It is a technique that systematizes for certain conditions the process of selecting the most desirable course of action, thereby giving management insight for making more effective decisions about the resources under its control [Ref. 1]

Many people know what linear programming is, but not all people know how to implement linear programming in the real world. Linear programming is not a complicated concept. Its basic argument is that when two or more things vary together, there may be a best combination for the two or more things, given a single goal like minimum cost or maximum yield. Almost all managers apply this notation in deciding on things like price levels (where price and units sold are related), or the best mix of products for a given production facility (where some mixes utilize capacity better than others), or the mix of compensation between base salary and incentives that will result, say in the highest sales (profits).

Linear programming problems are concerned with the efficient use and allocation of limited resources to meet a stated objective. These problems are characterized by a large number of solutions that satisfy the basic conditions of the problem. The selection of a particular solution depends upon an overall objective, such as maximizing profit or minimizing costs. A solution that both satisfies the conditions of the problem and accomplishes the objective is called an optimal solution. Finding this solution is known as a mathematical "programming problem."

[1]The term linear programming was suggested by T.C Koopmans in 1951 as an alternative to the earlier form, "programming in a linear structure."

The most difficult task is applying this simple idea to a complicated situation where many things go on simultaneously, and where a large fraction of all potential solutions may not work at all (are not feasible). The major contribution of linear programming is identifying problems where a best (optimum) feasible solution exist and providing a computational method for finding these solutions at a reasonable cost [Ref. 7].
Linear programming is one of a number of mathematical programming tools which seek a least-cost solution to meeting imprecise specifications. The distinction between linear programming and the other mathematical programming techniques is that linear programming requires linearly proportional relationships. The resources consumed by an activity must be linearly proportional to the level of the activity.

Linear programming is a specialized mathematical decision making tool. Each of us tries to make the best decisions possible with the tools available. Like other widely used mathematical methods, linear programming can only help us interpret data and to explore theories about the way things work or should work. If the data is spurious and incomplete, or if the mathematical model and theory are inappropriate, mathematical decision making aids such as linear programming are more likely to confuse rather than clarify a decision. Linear programming was developed for specific classes of applications which assume optimal behavior and have linear relationships [Ref. 8, pp. 3-15, 9, 10].

Finally, we must be able to implement an optimal solution if we hope to gain by applying linear programming to a problem. An optimal solution implies that decision makers are aware of all the important information on which to evaluate a decision, or at least can identify the best decision. There is one criterion that is best for all decisions. Some people regard economic gain as something to be maximized, others prefer to minimize the use of resources.

This thesis explains how linear programming methods work in actual practice, using a sequence of example problems from maintenance operations as the basis for discussion. Summarizing, linear programming is a mathematical tool for obtaining optimum solutions that do not violate imprecise constraints, that cannot have negative activity levels, that require linearly proportional relationships, and that account for all inputs and outputs within the system.

B. CLASSIFICATION OF PROGRAMMING PROBLEMS

Basically, there are two classes of programming problems, deterministic and probabilistic problems.

1. Deterministic Problems

The deterministic class implies that if certain actions are taken it can be predicted with certainty what will be the requirements to carry out those actions, and the outcome of any actions [Ref. 11, 12, 6]. I will use this class to solve the problem in our discussion in the following chapter.

2. Probabilistic Problems

Probabilistic problems are defined as programs involving uncertainty. Uncertainty can arise in many ways. For example, the outcome of a given action may depend on some chance event such as traffic delays, government policy, weather, employment levels, or the rise and fall of customer demand [Ref.11, 12, 6]. In the rest of this thesis we will not consider this class of models.

C. MODEL FORMULATION

1. Basic Concept

Some references use the term "model building" to express the procedures or the process of putting together mathematical symbols according to certain rules to form a structure (the model) which corresponds to certain aspects of a system in the real world.

From the 1940's to the 1990's, linear programming was applied to decision making in many fields such as economics, engineering, industry, and government because these areas contain many problems amenable to this type of model building. Also linear programming has the simplest mathematical structure which can be used to solve management problems.

As we shall see, linear programming is concerned with describing the relationship between the components in a system. T.C Koopmans introduced the term "activity analysis" to identify the elementary functions which make up a system [Ref. 6]. An activities is thought of as a kind of "black box"[2] into which flow tangible inputs, such as people, material, and equipment, and out of which flow tangible output such as the products of manufacturing, or the trained personnel of a military crew.

2. Basic Assumptions

To be a linear programming model, the system must satisfy certain assumptions of proportionality, nonnegativity, additivity and linearity [Ref. 6, 11].

a. Proportionality

The first assumption is proportionality. This means that in the linear programming model the amount of flow of various resources into and out of an activity are always proportional to the activity level. For example, if you wish to double an output resources, you simply double the activity level. The activity level needs to have a defined unit of measure. The resources need to also have defined units of measure [Ref. 11, 12, 9, 10, 6].

b. Nonnegativity

All activity levels have to be non-negative. In other words, negative activity levels are not possible. For example, we cannot ship a negative number of items from one place to another.

c. Additivity

Another characteristic of the linear programming model is the additivity assumption. A step in building a model is to specify that the system of activities be complete in the sense that a complete accounting of the activity can be made for each item. The total amount of each item specified by the system equals the sum of the amounts flowing into the various activities minus the sum of the amounts flowing out for each item. This is called a material balance equation [Ref. 6]. In other word, under this assumption the total amount of a resource used by the system equals the sum of the amounts of that resource used by the various activities.

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[2]Black box : any system whose detailed internal nature one willfully ignores [Ref. 6].

d. Linear Objective Function

The last characteristic of the linear programming model is known as the linear objective function assumption. One of the resources in the system is considered as "precious". It means that the total quantity of it produced by the system measures the payoff. The precious item could be completed assemblies, skilled labor, or an input resource that is in scarce supply like a limited monetary budget. The contribution of each activity to the total payoff is the amount of the precious resource that flows into or out of each activity. Therefore, if the objective is to maximize profits, activities that require money contribute negatively and those that money contribute positively to the total profits [Ref. 6].

3. Basic Model Components

The model components include **decision variables**, an **objective function**, and **model constraints.**

Decision variables usually are mathematical symbols that represent the unknown levels of activities being conducted by an organization. For example, suppose Indonesian Aircraft Industry (IPTN) desires to produce X1 C.250 aircraft, X2 C-235 aircraft, and X3 SA-330 aircraft. Then, X1, X2 and X3, are symbols representing unknown variable quantities of each activity (the producing of a certain aircraft). The final values of X1, X2 and X3 constitute a decision, e.g., X1 = 15 C-250's is a decision to produce 15 C-250's aircraft.

The objective function is a linear mathematical relationship of the decision variables that describes the objective of the organization in terms of the precious resources. Linear programming attempts to either maximize or minimize the objective function.

The model constraints are a result of the material balance equations described above and represent the restrictions placed on the organization by operating environment. The restrictions can be in the form of limited resources or restrictive guidelines. For example, only 250 days of labor may be available per year to produce aircraft. The actual numerical values in the objective function and the constraints, such as the 250 days of available labor, are parameters.

4. Building the Model

Building the model is the first and perhaps the most important step in the formulation of a linear programming problem. Dantzig provides a systematic procedure for building a linear programming model as follows [Ref. 6].

a. Define the Activities

Decompose the entire system under study into all of its elementary functions (activities) and choose a unit of measure for each activity by which its level can be quantified.

b. Define the Item Set

Determine the items (or resources) which are consumed or produced by the activities and choose a unit for measuring each item. Select one item such that the net quantity of it produced by the system as a whole measures the cost (the costs may happen to be money; however, in economic examples, they could be measured in terms of any scarce resource) of the entire system.

c. Determine the Input-output Coefficient

Determine the quantity of each item consumed or produced by the operation of each activity at its unit level. These quantities are called the input-output coefficients and are the factors of proportionality between activity levels and item flows.

d. Determine Exogenous Flows

Determine the net inputs or outputs of the items in the system taken as a whole. These values represent the bounds (or right-hand-side values) of the constraints (the material balance equations).

e. Determine the Material Balance Equations

Assign unknown nonnegativity levels to all activities. Then, for each item, write the material balance equation which asserts that the algebraic sum of the flows of that item into and out of each activity (given as the product of the activity level by the appropriate input-output coefficient) is equal to the exogenous flow of the item. This includes the materiel balance equation for the item which is considered "precious". That equation has an unknown exogenous flow since the equation will be the objective function for the model.

In developing the materiel balance equations for items other than the one used in the objective function the exogenous flows may not need to be met exactly; rather, they may be bounds on the items' total usage by all the activities. When this happens we state the equations as inequalities. Then, when the problem is being solved, slack and surplus variables are added to make the inequalities into equations because the solution procedure (the simplex method) only works with equations.

When we have ≤ inequality constraints, the solution software transforms ≤ inequality constraints into equations. This transformation is achieved by adding a new nonnegative variable, called a slack variable, to each constraint. For example, is

$2X_1 + 3X_2 + 2X_3 \leq 120$ hours of operation 1.

The addition of a unique slack variable s to an inequality results in the following equation;

$2X_1 + 3X_2 + 2X_3 + s = 120.$

If, in the optimal solution, the activity level values are $X_1 = 0$, $X_2 = 30$, $X_3 = 10$, substitution these values into the above equation yields:

$2(0) + 3(30) + 2(10) + s = 120$, or

$s = 120 - 110 = 10$ hours.

Thus, in general, s represents the amount of unused capacity in operation 1.

Instead of adding a slack variable to a ≥ constraint, the solution software subtracts a nonnegative surplus variable. Whereas a slack variable reflects the amount of unused resource, a surplus variable reflects the excess above a minimum resource requirement level. Like the slack variable, a surplus variable can be represented symbolically by s.

For example, the dual linear programming problem is

$2Y_1 + 4Y_2 + 3Y_3 + 1Y_4 \geq 40$

The subtraction of a nonnegative surplus variable gives:

$2Y_1 + 4Y_2 + 3Y_3 + 1Y_4 - s = 40$

If the optimal values are $Y_1 = 0$, $Y_2 = 0$, $Y_3 = 5$, and $Y_4 = 25$, then substituting these values into above equation yields:

$2(0) + 4(0) + 3(5) + 1(25) - s = 40$

$s = 40 - 40 = 0.$

In this case there is no excess resources, assuming the constraint is a resource constraint of some type.

Once the materiel balance equations or inequalities have been developed, the linear programming problem can be posed in mathematical terms; that is, determine levels for the activities X_1, X_2, ...X_8 which (a) are nonnegative (b) satisfy the material balance equations, and (c) minimize or maximize the objective function. [Ref. 6] The solution can be interpreted as a program for the system - a statement of the time and quantity of actions to be performed by the system so that it may move from its current status toward the defined objective.

f. Solve for Optimal Values of Variants (feasible solution)

The linear programming problem is to find the optimal feasible values of the decision variables that will maximize the total profit or minimize the total cost (the objective function subject to the constraints on resources and the nonnegativity conditions). A feasible solution is solution that satisfies the constraints and nonnegativity conditions. Linear programming problems typically have more decision variables (unknown activity levels) then there are material balance equations. As a consequence, a solution to this type of system of linear equations will only have positive values for at most a number of decision variables equal to the number of equations. The positive-valued decision variables are called dependent or basic variables and the rest are called independent or non-basic variables. The reason for calling the positive variables "dependent" is that their values are determined after setting the independent variables' values to zero. A basic feasible solution is defined as the special solution obtained by setting the independent (non-basic) variables equal to zero and solving for the nonnegative values of the dependent (basic) variables [Ref. 6].

Historically, the simplex method has been the technique for solving linear programming problems. In the simplex method, the model is put into the form of a table and then a number of mathematical steps are performed on the table. These mathematical steps are the process for moving from one basic feasible solution to the next until the optimal solution is reached. One iteration of the simplex method uses the information from the current table to generate a new table corresponding to the new basic feasible solution.

D. POST-OPTIMALITY ANALYSIS

Post-optimality analysis means the analysis of the optimal basic feasible solution in order to gather additional information [Ref. 11]. The optimal solution of a linear programming model can be analyzed in two ways: sensitivity analysis and duality analysis.

1. Sensitivity Analysis

When a linear programming model is formulated, it is implicitly assumed that the parameters of the model are known with certainty. These parameters include the objective function coefficients, such as profit per unit of an activity; model constraint quantity values, such as the total available hours of labor; and constraint coefficients (input-output coefficients), such as pounds of material used to produce an item. However, rarely does a manager know all of these parameters exactly. In reality, the model parameters are simply estimates or "best guesses". For this reason it is of interest to the manager to see what effect a change in a parameter may have on the solution to the model. The analysis of the effects on the model solution of making a small change in the value of a parameter is known as sensitivity analysis.

For example, suppose we have an estimate of the absentee rate of our labor force during the next month, and the model has been run using the estimate. What happens to the optimal solution if we change the estimate by 5%, 10% or even 15%? Will the optimal objective function value vary widely, or will it remain more or less unchanged? Obviously, the answer to such questions will help to determine the credibility of the model's recommendations. For example, if the objective function value changes very little with large changes in the value of a particular parameter, we will not be concerned about uncertainty in the current value of that parameter. If, on the other hand, the objective function value varies widely with small changes in that parameter, we cannot tolerate much uncertainty in its value.

The most obvious way to ascertain the effect of a change in the parameter of a model is to make the change in the original model, resolve the model, and compare the solution results with the original. However, resolving a problem numerous times for all possible combinations of changes can be very time consuming. In most cases the effect of minor changes on the model can be determined directly from the final simplex table (commonly called a tableau) [Ref. 11, pp. 145-147]. We, however, will use the ABQM program output instead to interpret and analyze the optimal simplex tableau. ABQM is a management science software package published by Allyn and Bacon [Ref.11]. It will automatically provide the sensitivity analyses for a given problem.

Making good use of computer analysis is, of course, a problem faced by managers in the real world. In this analysis we lay some of the groundwork for being able to understand clearly the meaning of the computer results from ABQM software.

a. Changes in Objective Function Coefficients

It can be demonstrated that a change in one of the coefficients of the objective function may change the optimal solution. Therefore, sensitivity analysis is performed to determine the range over which each coefficient of the objective function can be changed, assuming the others are not, without altering the optimal solution.

The result is a range of values for a coefficient of the objective function that will not result in a change in the optimal solution. If the coefficient is associated with a non-zero basic variable in the optimal solution, then the objective function's optimal values will change linearly as the coefficient is varied even though the solution does not. If the coefficient is associated with a non-basic basic variable (which will always have a value of zero) or a basic variable having a zero value (i.e., a degenerate optimal solution) then the objective function's optimal value will not change.

b. Changes in Constraint Quantity Values

A change in the total amount of a constrained resource may change the feasible solution. If the constraint is not binding (i.e., the resource associated with that constraint is not completely used up in the optimal solution) then the value of the slack variable (it will be part of the basic feasible solution output) is the amount that the total quantity of the resource can change before affecting the optimal solution. If the constraint is binding, then any change in the total quantity of the resource will change the optimal solution.

c. *Changes in the Constraint Coefficients (Input-Output Coefficients Change)*

Changing the input-output coefficients can alter the optimal solution if the coefficient is associated with a non-zero basic variable. The entire basic feasible solution usually changes. The impact is more complex than that for the objective function's coefficients or the resource total quantities (constraint quantity values). The coefficients for non-basic variables and basic variables having zero value can vary without changing the optimal solution.

2. Duality Analysis

The duality analysis is a method based on the dual model of linear programming. The dual model is derived completely from the primal model. This model provides decision makers with an alternative way of looking at a problem. Because the dual model provides information regarding the economic value of the constrained resources it is useful for examining ways of increasing profit or decreasing cost [Ref. 11].

Table 1 illustrates how the dual model is derived from the primal.

Primal	Dual
$Max{=}Z_P = 40\ X_1 + 35\ X_2 + 45\ X_3$	$Min{=}Z_d = 120\ Y_1 + 160\ Y_2 + 100\ Y_3 + 40\ Y_4$
subject to	subject to
$2\ X_1 + 3\ X_2 + 2\ X_3 \leq 120$	$2\ Y_1 + 4\ Y_2 + 3\ Y_3 + \ Y_4 \geq 40$
$4\ X_1 + 3\ X_2 + \ X_3 \leq 160$	$3Y_1 + 3\ Y_2 + 2\ Y_3 + 1\ Y_4 \geq 35$
$3\ X_1 + 2\ X_2 + 4\ X_3 \leq 100$	$2\ Y_1 + \ Y_2 + 4\ Y_3 + \ Y_4 \geq 45$
$X_1 + \ X_2 + \ X_3 \leq 40$	
$X_1, X_2, X_3 \geq 0$	$Y_1, Y_2, Y_3 \geq 0$

Table 1. The Primal-dual Relationships.

From Table 1 the following relationships between the primal and dual model can be seen [Ref. 11, 6, 13, 15] :

- The dual variables Y_1, Y_2, Y_3, and Y_4, correspond to the model constraints in the primal. For every constraint in the primal there will be a variable in the dual. For example, in this case the primal has four constraints; therefore, the dual has four decision variables.

- The objective function coefficients in the primal model, 40, 35, and 45 represent the model constraint requirements (quantity values on the right-hand side of the constraints) in the dual.
- The model constraint coefficients in the primal are also the decision variable coefficients in the dual. For example, coefficients in the first constraint are 2, 3, and 2. These values are the Y_1 variable coefficients in the model constraints of the dual: $2Y_1$, $3Y_1$, and $2Y_1$.
- The quantity values on the right-hand side of the primal inequality constraints are the objective function coefficients in the dual. The constraint quantity values in the primal; namely, 120, 160, 100, and 40, are used to form the dual objective function : $Z_d = 120Y_1 + 160Y_2 + 100Y_3 + 40Y_4$.
- The maximization primal model has $\leq$ constraints, the minimization dual model has $\geq$ constraints. The converse is also true.

The optimal solution to the dual provides values for the Y's. These are called "shadow prices" associated with the constraint quantity values of the primal model. As noted above, each primal constraint has one Y variable associated with it. The value of Y for a given constraint tells the decision makers how much the optimal primal model's objective function value will change with a unit change in the right-hand side value of the constraint. For example, suppose $Y_3 = 5$ in the dual optimal solution. Then that suggests that the optimal value of the primal objective function will increase by 5 for an increase in the third constraint's right-hand side from 100 to 101. It is important to note that this is really a marginal analysis and that an actual change in the right-hand side may give a different change in the objective function.

E. HOW CAN LINEAR PROGRAMMING BE USED IN RESOURCE ALLOCATION DECISION MAKING?

Linear programming problems are concerned with the efficient use or allocation of limited resources to meet desired objectives. These problems are characterized by the large number of solutions that satisfy the basic conditions of each problem. The selection of a particular solution as the best solution to a problem depends on some aim or overall objective that is implied in the statement of the problem. As mention above, solution that satisfies both the constraint conditions of the problem and the given objective is termed an optimal solution. A typical example is that of the manufacturing company that must determine what combination of available resources will enable it to manufacture products in a way which not only satisfies its production schedule, but also maximizes its profit or, in the case of a military organization maximizes its readiness. at maximum readiness.

In 1947, Dantzig initiated the development of linear programming modeling with inequality constraints and invented the simplex method. Shortly after World War II, a group of scientists was called upon by the U.S. Air Force to investigate the feasibility of applying mathematical techniques to problems of military logistics that were of particular concern. George Dantzig was one member of the research team. Dantzig had earlier proposed that the interrelations between activities of a large organization be viewed as a linear programming model and that the optimal program (or selection) be determined by minimizing a (single) linear objective function. Such ideas led the Air Force to set up a team under the project name SCOOP (Scientific Computation of Optimum Programs).

By the middle of 1947 Dantzig and his associates had developed not only an initial linear programming mathematical model but also a general method for solving linear programming problems. This method was called the "Simplex Method." [Ref. 16]

As a consequence of these developments, linear programming was quickly adopted by mathematicians, economics, operations researchers, and individuals in a number of military and civilian organizations. Some examples of successful linear programming in aviation are discussed below:

1. U.S. Air Force Staff

A linear programming model is used to help the U.S. Air Force Staff decide how much to spend on different types of aircraft and munitions. Currently this model is being used by the Air Force Center for Studies and Analyses and is being tested by the Munitions Division of the Plans and Operations Directorate (AF/XOXFM) for munitions trade-off analyses. The Air Force model uses existing data estimates on aircraft and munitions effectiveness, target value, attrition, aircraft and munitions cost, and existing inventories of aircraft and munitions. Other factors considered are weather and the length of conflict. Enhancements currently being implemented include attrition changes as a function of time and the distance the aircraft must fly. Decision variables are the total number of sorties flown by each aircraft/munitions combination against each target type in each of six different weather bands. The model includes the probability of killing target types under the six conditions of weather and the subjective value of target types. The model constraints include number of available aircraft and munitions, number of targets, weather conditions, and budget [Ref. 17].

2. Delta Airlines and MAC

Delta Airlines is one of the biggest companies in the United States and became the first commercial airline to use Karmarkar's program, called KORBX, developed by Narendra Karmarkar and sold by AT&T. The simplex method finds an optimal solution by moving from one adjacent corner point to the next, following the outside edges of the feasible region [Ref. 6,12]. In contrast, Karmarkar's method follows a path of points on the inside of the feasible region. Its uniqueness is its ability to handle an extremely large number of constraints and variables [Ref. 12, 18, 19].

Delta Airlines found that the Karmarkar program streamlined the monthly scheduling of 7,000 pilots who flew more than 400 airplanes to 166 cities world-wide. With increased efficiency in allocating limited resources, Delta thinks it will save millions of dollars in crew time and related costs.

Another user is the U.S. Air Force Military Airlift Command (MAC). Prior to the arrival of KORBX, MAC's linear programming problem was too big to run on one computer. Even a scaled-down version of the problem had 36,000 variables and 10,000 constraints and took four hours with simplex-based linear programming software on a mainframe. Now, models that include the entire, previously unsolvable Pacific Ocean system run in just 20 minutes on KORBX [Ref. 12].

3. American Airlines

Linear programming techniques have a direct impact on the efficiency and profitability of major airlines. Thomas Cook, President of American Airlines Decision Technology Group, tells us why optimal solutions are essential in his industry.

Finding an optimal solution means finding the best solution. Let's say you are trying to minimize cost or a cost function of some kind. For example, we may want to minimize the excess cost related to scheduling crews, hotels, and other costs that are not associated with the flight line and considering constraints, such as the amount of time a pilot can fly, how much rest time is needed, and so forth.

An optimal solution, then, is either a minimum-cost solution or maximizing solution. For example, we might want to maximize the profit associated with assigning aircraft to the schedule; so we assign large aircraft to high-need segments and small aircraft to low-load segments. Whether it's a minimum or maximum solution depends on what function we are trying to optimize.

Finding fast solutions to linear programming problems is also essential. A good example is a major weather disruption. If we get a major disruption at one of the hubs, such as Dallas or Chicago, then a lot of crews and airplanes are in the wrong place. What we need is a way to put that whole operation back together again, so that the crews and airplanes are in the right place. That way we minimize the cost of the disruption and minimize the passenger inconvenience. [Ref. 12]

4. UH-1 Helicopters

Kimberly S. Schenken, an operation research system analyst in the U.S. Army Aviation System Command (AMSAV-BD), used linear programming to solve this problem associated with the engine for the UH-1 helicopter [Ref. 20].

"In making decisions, the U.S. Army had conflicting goals:

(1) Minimize Total Life Cycle Cost. Minimizing total life cycle cost means minimizing the investment cost and the operating and support costs. However, there may be a conflict between minimizing investment costs and operating and support costs. The least expensive engine was probably not the least expensive to operate and support.

(2) Maximize Performance of the Engine. The U.S. Army wanted the highest performance engine since such an engine would increase the war fighting capability of the aircraft. Also, it would increase crew safety and avert possible loss as increased reserve engine power decreases the risk of crash and possible loss of life and property".

Reference 18 describes in detail the nature of the problem, the goals and trade-off involved in its possible solutions. It demonstrates how goal programming was useful in bringing order to the decision making process so all important aspects of the problems were considered. The reference also discusses the limitations of goal programming and the problems encountered in formulating the problem. By applying linear programming the U.S. Army expects to reduce the helicopter engine's total life cycle costs from $5,179,676 to $4,416,296 [Ref. 21].

F. SUMMARY

This chapter has presented the background theory needed for subsequent discussion of applying linear programming to Indonesian Air Force aircraft maintenance activities. After determining the benefit desired, the objective function can either be maximized or minimized. In civilian firms, the linear programming model is used to maximize profit or minimize the cost of labor or other scarce resources. In military organizations the model should optimize readiness or minimize life cycle costs. Some examples of real world problems in both civilian and military aviation were presented. Those examples show that linear programming can help a decision maker to make the best possible use of available resources. See Appendix A for two additional examples of linear programming applications in non-aviation organizations.

III. A LINEAR PROGRAMMING EXAMPLE

This chapter presents an example of applying the linear programming approach to an avionics maintenance problem. The application begin with a model formation and ends with a sensitivity analysis of the optimal solution. This chapter also suggests potential systems in the Indonesian Air Force avionics maintenance organizations which would benefit from the implementation of linear programming.

A. THE EXAMPLE

1. The Maintenance Problem

The main reason for selection of the area of avionics maintenance for analysis of linear programming benefits is based on the fact that there are large numbers of electronic and avionics equipment that are repaired each year. These numbers will increase even further in the next five years because of the recent purchase of a large number of new transport and fighter aircraft by the Indonesian Air Force. In addition, very limited resources must be allocated among the various products that Avionics Shops produce.

The Avionics Shop's overall objective is to maximize readiness (given that a specified amount of avionics equipment must be produced, how much money can be saved). The amount of money that can be saved typically is the difference between the cost of one piece of equipment being repaired outside under a maintenance contract and the cost of repairing it by the Air Force's maintenance organization. A maintenance contract costs more per unit repaired than if it is done in-house by the Air Force's maintenance organization.

This organization's primary mission is to maintain and repair highly technical products, communication, airborne radar, and guidance equipment. This equipment is used to support fighter and transport aircraft. Each piece of equipment that is repaired in the Air Force's avionics maintenance operation must pass through the following steps:

- Receiving/physical inspection and writing up a work order (Phase 1).
- Pre-inspection and fault diagnosis using ATE (Automatic Test Equipment) (Phase 2).
- Repair (Phase 3).
- Final inspection (Phase 4).
- Documentation is part of the phases 1,2,3 and 4. Shipping consists of a user or the air base picking up the repaired equipment from the avionics maintenance shops, so it can be ignored in the model to developed.

The flow chart of the avionics maintenance process is diagrammed as Figure 1.

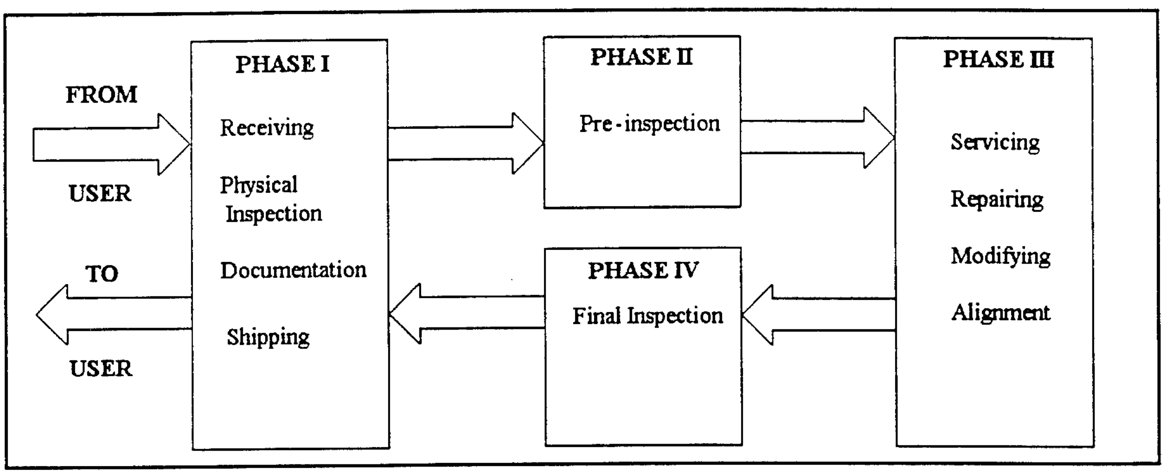

Figure 1. Flow Chart of Maintenance Operations.

The time required (in hours) for repairing a unit of avionics equipment is summarized in Table 2.

The capacity available in the maintenance shops available (in hours per year) for each Phase and the minimum annual production requirement to fulfill the Air Force's operational requirements are shown in Table 3. The details the capacity determinations are given in Appendix D.

Equipment	Phase 1	Phase 2	Phase 3	Phase 4
Comm	.25	.25	5	.75
Nav	.25	.25	6	.75
Airborne Radar	.25	.50	7	1.5
Guidance	.25	.50	8	1.5

Table 2. Time Required For Repairing of Each Unit of Equipment (hours).

Phase	Annual Capacity (hours)	Product	Min Annual Production (units)
1	1,540	Communication	236
2	770	Navigation	205
3	52,360	Airborne Radar	155
4	770	Guidance System	144

Table 3. The Capacity Hours Available and the Minimum Production Required Per Year.

2. Maintenance Contract Costs

Before the Air Force avionics maintenance facilities were established, the Indonesian Air Force had avionics equipment repaired outside the Air Force through a maintenance contract with a private vendor. However, it took more "turn around time" (from delivery until return of the equipment) to have equipment repaired and was more expensive than the Air Force considered acceptable. Based on rough data during last few years of contracting (1995-1991), the average cost of a "maintenance contract" for each type of equipment can be summarized as shown in Table 4 .

Equipment	Average
Communication	$ 4,750
Navigation	$ 4,750
Airborne Radar	$ 8,000
Guidance System	$ 24,000

Table 4. Average Cost of "Maintenance Contract" for Each Unit of Equipment.

3. The Air Force's Maintenance Costs

During the ten years of the existence of avionics maintenance in the Air Force, it has repaired a lot of avionics equipment. Having the Air Force perform the maintenance has some advantages:

- Less turn around time;
- Reduced maintenance cost; and
- Increased personnel knowledge.

The equipment being repaired requires spare parts (bits and pieces and modules) to replace the unserviceable components. The cost of spare parts and labor needed for each type equipment being repaired varied depending on the nature of the failures. Based on my experience, the average total cost (spare parts' cost and labor cost) of the avionics equipments are given in Table 5 and are rough estimates using data from 1985-1991 [Ref. 19,22].

Equipment	Average
Communication	$ 1,500
Navigation	$ 1,250
Airborne Radar	$ 2,250
Guidance System	$ 7,750

Table 5. Average Cost of the Repair of Each Unit of Equipment by the Air Force.

B. FORMULATING THE PROBLEM

1. Building the Model

To formulate the linear programming problem we next apply the steps that were described in Chapter II.

a. Step 1 - Define Activities

The four activities in the Air Force's maintenance operation are:

- Repair communication equipment (measured in units per year).
- Repair navigation equipment (measured in units per year).
- Repair radar equipment (measured in units per year).
- Repair guidance equipment (measured in units per year).

Similarly, the four activities in a maintenance contract are:

- Repair communication equipment (measured in units per year).
- Repair navigation equipment (measured in units per year).
- Repair radar equipment (measured in units per year).
- Repair guidance equipment (measured in units per year).

b. Step 2 - Define the Item set

These can be readily deduced from Figure 1 and Table 2.

- Capacity in receiving, visual check, and work order (measured in hours)
- Capacity in pre-inspection check (measured in hours)
- Capacity in the repair process (measured in hours)
- Capacity in final check (measured in hours)
- Cost (measured in dollars)

c. Step 3 - Input-Output Coefficients

Input-output coefficients for the four activities are shown in Figure 2 through 9. These are obtained, input, from Table 2, Table 3, and Table 4. Note that contracting activities do not consume any Air Force maintenance capacity in doing repair.

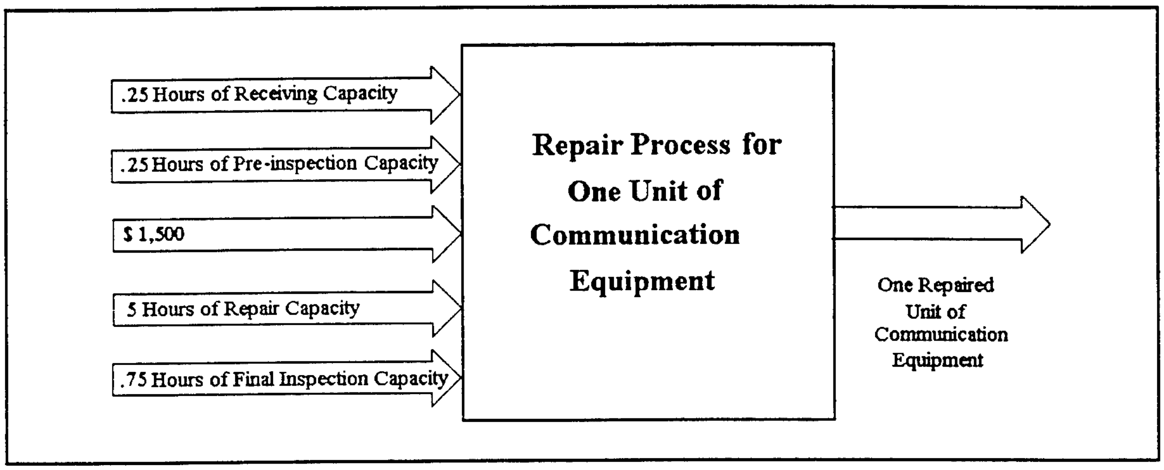

Figure 2. Repair Process for One Unit of Communication Equipment.

d. Step 4 - Exogenous Flows

Since the capacities in each of the four phases (receiving, pre-inspection, repair, and final inspection) are inputs to each of these activities, they must be inputs to the system as a whole. The total required output from repair are exogenous flows corresponding to the number of units needed to be output from each activity. Table 3 provides values for both these types of exogenous flows.

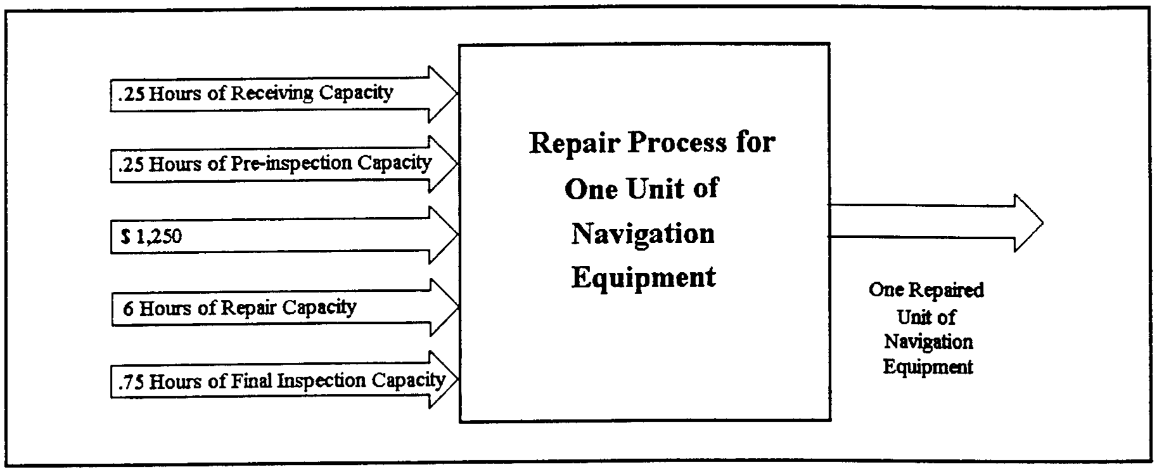

Figure 3. Repair Process for One Unit of Navigation Equipment.

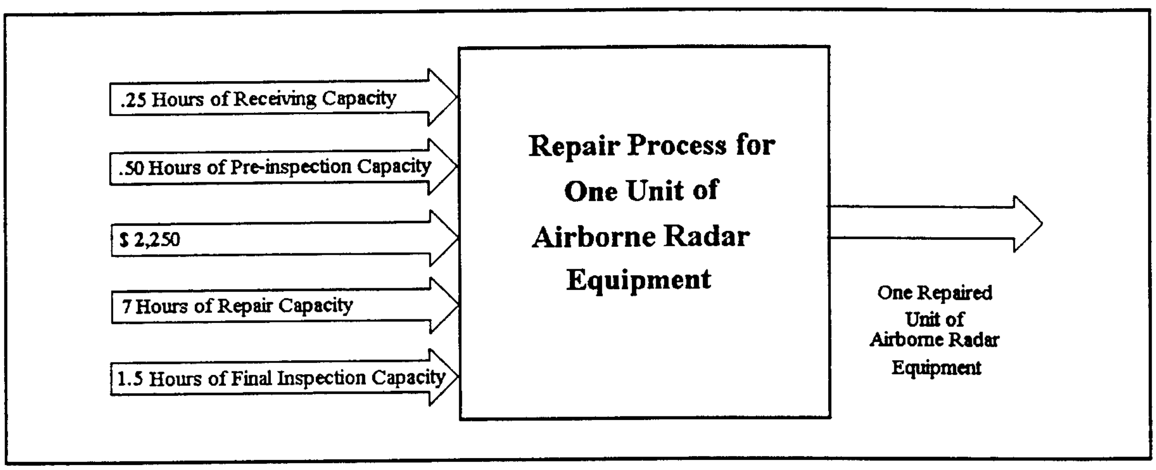

Figure 4. Repair Process for One Unit of Airborne Radar Equipment.

e. Step 5 - Material Balance Equations

The first step is to assign an unknown activity level to each activity.

X_1 = Number of units of communication equipment repaired by the Air Force per year;

X_2 = Number of units of navigation equipment repaired by the Air Force per year;

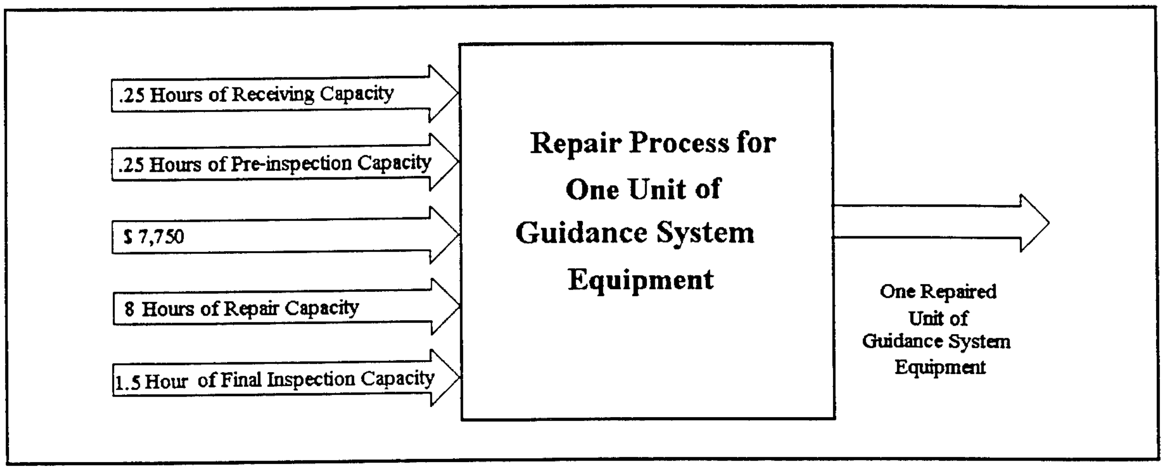

Figure 5. Repair Process for One Unit of Guidance System Equipment.

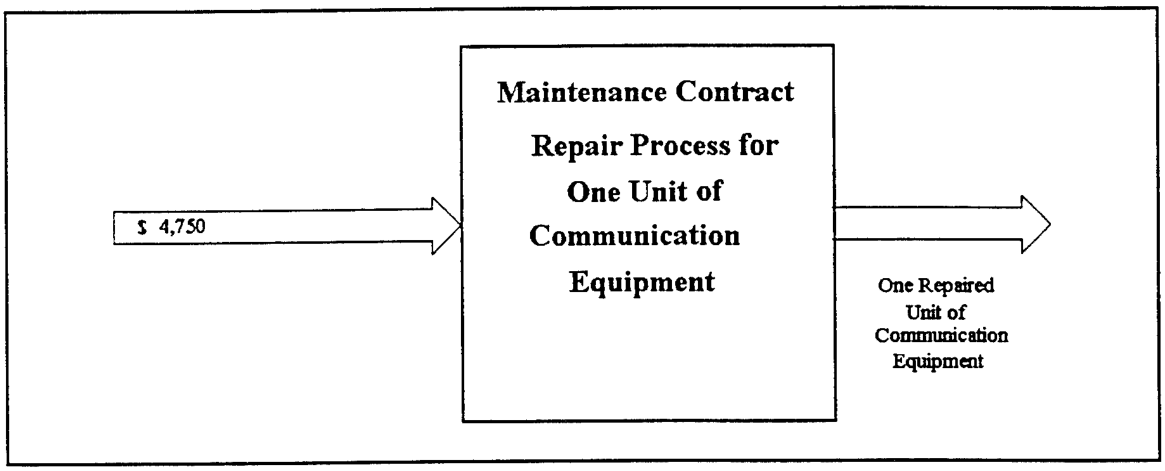

Figure 6. Repair Process for One Unit of Communication equipment (Contract).

X_3 = Number of units of airborne radar equipment repaired by the Air Force per year;

X_4 = Number of units of guidance system equipment repaired by the Air Force per year;

X_5 = Number of units of communication equipment repaired by contract per year;

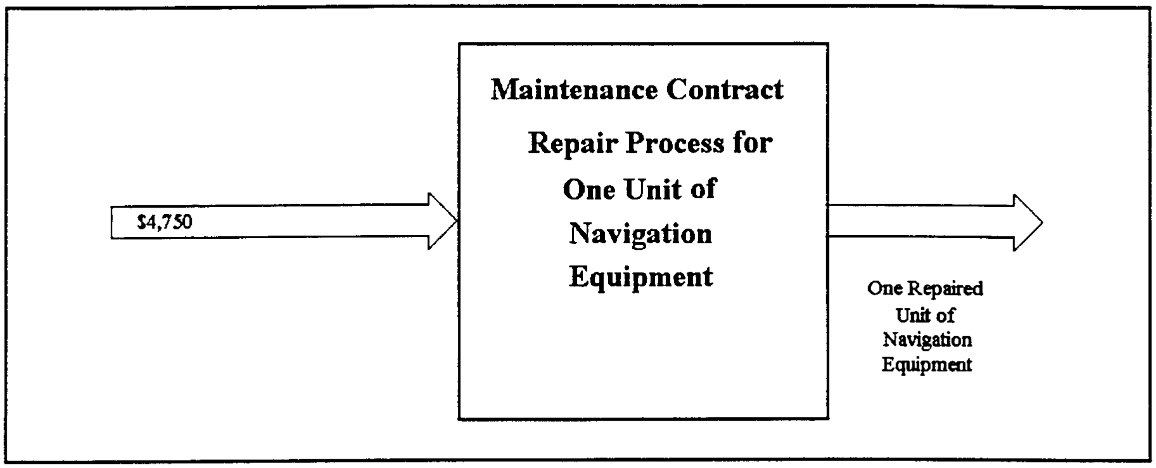

Figure 7. Repair Process for One Unit of Navigation Equipment (Contract).

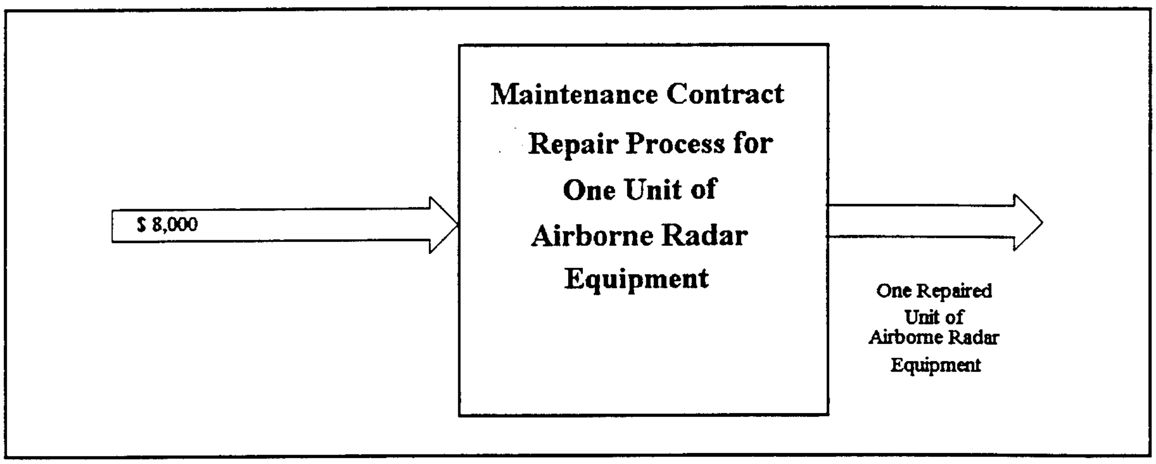

Figure 8. Repair Process for One Unit of Airborne Radar Equipment (Contract).

X_6 = Number of units of navigation equipment repaired by contract per year;

X_7 = Number of units of airborne radar equipment repaired by contract per year; and

X_8 = Number of units of guidance equipment repaired by contract per year.

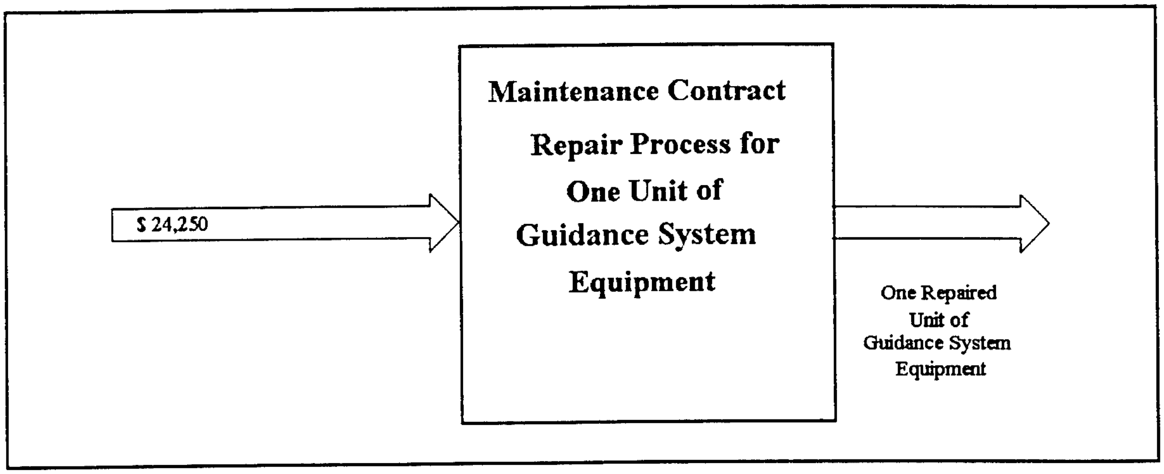

Figure 9. Repair Process for One Unit of Guidance Equipment (Contract).

Next, we write material balance equations. Since we may not use up the available hours of phase capacity and we may exceed production minimums, we will write the balance equations in inequality form. For the receiving, visual check, and work order phase we get

$$.25X_1 + .25X_2 + .25X_3 + .25X_4 \leq 1540 \text{ hours.}$$

The "pre-inspection check" constraint is:

$$.25X_1 + .25X_2 + .50X_3 + .50X_4 \leq 770 \text{ hours.}$$

The "repairing " constraint is:

$$5X_1 + 6X_2 + 7X_3 + 8X_4 \leq 52{,}360 \text{ hours.}$$

The "final inspection" constraint is:

$$.75X_1 + .75X_2 + 1.5X_3 + 1.5\,X_4 \leq 770 \text{ hours.}$$

Next, we write the output constraints. As noted above, the constraints on the numbers of avionics equipment which are to be repaired per year are also in the inequality form. The "communication repair" constraint is:

$$-X_1 - X_5 \leq -236 \text{ units.}$$

The reason for all the minus signs is that outputs are being considered only and the convention given by Reference 6 is that outputs have a minus sign in the material balance equations and that inputs have a plus sign. We will later change these signs by multiplying both sides by -1.

The "navigation repair" constraint is:

$$- X_2 - X_6 \leq - 205 \text{ units.}$$

The "airborne radar repair" constraint is:

$$- X_3 - X_7 \leq - 155 \text{ units.}$$

The "guidance system repair" constraint is:

$$- X_4 - X_8 \leq - 144 \text{ units.}$$

Finally, the objective function is the material balance of costs input to each activity.

$$1500x_1 + 1250x_2 + 2250x_3 + 7750x_4 + 4750x_5 + 4750x_6 + 8000x_7 + 24250x_8 = + Z.$$

We let Z denote the unknown total cost. Our interest is in finding a feasible solution which minimizes the value of Z. Finally, we need to specify that no activity level is allowed to be negative.

Therefore, $X_1 \geq 0$, $X_2 \geq 0$, $X_3 \geq 0$, $X_4 \geq 0$,

$X_5 \geq 0$, $X_6 \geq 0$, $X_7 \geq 0$, $X_8 \geq 0$.

This completes the model formulation.

f. Step 6. The Problem Statement

The linear programming model has been completely formulated and the problem can now be formally stated as: Find values of X_1, X_2, X_3, X_4, X_5, X_6, X_7, and X_8 which

$$\text{Minimize } Z = 1500X_1 + 1250X_2 + 2250X_3 + 7750X_4 + 4750X_5 + 4750X_6 + 8000X_7 + 24250X_8$$

subject to:

$$.25X_1 + .25X_2 + .25X_3 + .25X_4 \leq 1540$$

$$.25X_1 + .25X_2 + .50X_3 + .50X_4 \leq 770$$

$$5X_1 + 6X_2 + 7X_3 + 8X_4 \leq 52360$$

$$.75X_1 + .75X_2 + 1.5X_3 + 1.5X_4 \leq 770$$

$$X_1 + X_5 \geq 236$$

$$X_2 + X_6 \geq 205$$

$$X_3 + X_7 \geq 155$$

$X_4 + X_8 \geq 144$

$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8 \geq 0.$

C. OPTIMAL SOLUTION AND ANALYSIS

This section presents the solution and sensitivity analysis of the linear programming problem of the avionics maintenance operations of the Air Force that was formulated in Section C. ABQM software was used to solve the problem. The user is required to enter the data, decision variables, objective function, and model constraints into the program. ABQM software then uses the simplex method to solve the problem.

1. Solution

Appendix B shows the ABQM software required inputs and program outputs for the problem formulated in the preceding section. The optimal solution of the linear programming is listed below. Appendix B provides this solution at the bottom of the first page and top of the second page.

X_1 = 236 units,

X_2 = 205 units,

X_3 = 148.833 units,

X_4 = 144 units,

X_5 = 0 units,

X_6 = 0 units,

X_7 = 6.167 units,

X_8 = 0 units, and

Z = $ 2,110,458.333,

where:

X_1 = Number of units of communication equipment repaired by the Air Force per year.

X_2 = Number of units of navigation equipment repaired by the Air Force per year.

X_3 = Number of units of airborne radar equipment repaired by the Air Force per year.

X_4 = Number of units of guidance equipment repaired by the Air Force per year.

X_5 = Number of units of communication equipment repaired by contract.

X_6 = Number of units of navigation equipment repaired by contract.

X_7 = Number of units of airborne radar equipment repaired by contract.

X_8 = Number of units of guidance equipment repaired by contract.

Finally,

Z = The total cost of the maintenance (by the Air Force and by contract) per year.

In the optimal solution the value of X_7 is greater than zero. It means the Air Force has to "contract out" for repair of communication equipment. This is because there is insufficient capacity in the final inspection phase. Constraint C_4 is the only constraint in phase capacity which is binding. The table in the middle of the second page of Appendix B shows that its slack variable is zero. Constraints C_1, C_2, and C_3 are not binding since the values of their slack variables are 1356.542, 513.333, and 47756.167, respectively. Finally, constraints C_5, C_6, C_7, and C_8 are binding since their slack variables are zero.

Decision variables X_1 through X_4, and X_7 are the basic variables (their values are positive) and X_5, X_6, and X_8 are non-basic variables (their values are zero) in this optimal solution.

2. Analysis

Both a sensitivity analysis and duality analysis are also provided in Appendix B.

a. Sensitivity Analysis

(1) Objective function coefficient ranges. Ranges are provided at the bottom of the second page of Appendix B. The objective coefficient range for X_3 (airborne radar equipment) is from $1,500 to $8,000. The current value is $2,250 so the allowable increase is $5,750 and allowable decrease is $750 before the optimal solution would change. However, the objective function value changes linearly if the cost coefficient for X_3 changes based on the optimal value of X_3. For example, if the objective

function's cost coefficient for X_3 changes from $2,250 to $2,249, the objective function value will change from $2,110,458.333 to $2,110,309..5, a difference of $148.833 since X_3's value is 148.33 and its cost coefficient was reduced by one unit.

The current value of X_1 is 236, and its cost coefficient's allowable increase is $375, and the allowable decrease is $4,375. If, for example, the cost coefficient for X_1 changes from $1,500 to $1,449, the value of the objective function will be reduced by $236.

The current value of X_2 is 205, its cost coefficient is $12,250. The allowable increase in the cost coefficient is $625 and its allowable decrease is $4,125. If the cost coefficient for X_3 changes from 2,250 to 2,249, the objective function value will be reduced by $155.

The current value of X_4 is 144 and its cost coefficient's allowable increase is $10,750 and allowable decrease is $13,500 If the cost coefficient for X_4 changes from $7,750 to $7,749, the value of the objective function will be reduced by $144.

The current value of X_7 is 6.167 and its cost coefficient's allowable increase and decrease are $750 and $5,750, respectively. If the cost coefficient for X_7 changes from $8,000 to $7,999, the value of the objective function will be reduced by $6.167.

The remaining variables, X_5, X_6, and X_8, all are currently zero. Thus, any change in their cost coefficients will not change the value of the objective function. However, if any of the cost coefficients are reduced below the allowable decrease, then a new optimal solution will result which will include the variable who cost coefficient has been so reduced. Notice that the optimal solution table shows a column called "Reduced Costs." The reduced costs are zero for X_1 through X_4, and X_7 meaning any cost coefficient change will result in a change in the objective function value. The reduced costs for X_5, X_6, and X_8 are identical to the allowable decrease values.

In summary, this analysis shows that if we change the cost coefficients within the allowable increase and decrease range of values for the positive decision variables the objective function value will change. However, no change will result from changing the cost coefficients for the decision variables which are zero. If we change any cost coefficients beyond the allowable increase and decrease range, the optimal solution will change and so will the value of the objective function.

(2) Right hand side ranges. The capacity constraints are labor hour constraint (constraints 1 through 4). The next four constraints are lower bounds or the number of units required to be repaired each year to meet readiness goals. We already know from the table following the optimal solution that the first three capacity constraints are slack and the rest of the constraints have no slack (are binding). Therefore we can reduce the right-hand side values of the first three constraints by the amount of the slack variables before the optimal solution will change. On the other hand, any change in the right-hand side values of the binding constraints will result in a change in the optimal solution. When we examine the table called "Right Hand Side Ranges" which is at the top of the third page in Appendix B, we see that allowable decreases for the first three constraints are exactly equal to the slack variable values. What about the information in that table for the binding constraints (those which aren't slack)? The information tells us how much the right-hand side value for any binding constraint will need to change before a new optimal basic feasible solution will replace the current one. Any change of less value will only result in changes in the values of the current basic variables.

b. Duality Analysis

In Chapter III we discussed the relationship between the primal and dual models and said that the solution to the dual gives us an indication of the marginal change in the optimal objective function value of the primal problem for a unit change in the right-hand sides of the constraints. The ABQM software provides this information as part of the output. For our problem the table in the middle of the second page of Appendix B not only gives the values of the slack variables for each constraint but also the solution to the dual problem which we referred to in Chapter III as the "shadow prices." From this table in Appendix B we see that the first three constraints have zero shadow prices. Therefore, a unit change in any of their right-hand side values will result in no change in the optimal solution and, as a consequence, no change in the objective function value. Constraints C_4 through C_8, on the other hand, are binding and any change in the right-hand sides will result in a change in the optimal solution. An estimate of the impact of a unit change in any right-hand side value on the value of the objective function is provided by the shadow prices. For example, if the right-hand side of the constraint C_4 is reduced from 770 to 769, the optimal value of the objective function is expected to increase by \$3,833.33. Similarly, if it is increased from 770 to 771, the objective function value is expected to decrease by \$3,833.33. Since this constraint has 770 as its upper bound, an increase to 771 is less restrictive than 770 and therefore a cost reduction is expected.

For constraints C_5 through C_8, the right-hand values are lower bounds. Reducing any one of those bounds will therefore result in a marginal reduction in the objective function's value and any increase will increase the objective function's value. For example, if the right-hand side of constraint C_5 is decreased from 236 to 235, the marginal reduction in the objective function is \$4,375 (the shadow price is -4,375) while an increase on the right-hand side value from 236 to 237 is expected to increase the objective function's value by \$4,375. The biggest marginal improvement in the objective function occurs for a decrease in the right-hand side value of constraint C_8. If its value is reduced from 144 to 143, then marginal change in the objective function will be a decease of \$13,500. Thus, in attempting to determine which right-hand side values are critical, it is sufficient to look at their shadow prices. The ones that are critical should then be reviewed to see if they are accurate or can be changed to reduce the optimal total costs.

The shadow prices do not say how much of a change in a right-hand side is possible before the marginal savings are no longer valid. How the basic variables' values change for even a one unit change in the right-hand side is not provided by the shadow prices. To find out how these variables' values change requires resolving the problem with the change in the right hand side. Appendix C illustrates the results when a unit change is made to the right-hand side of constraints C_5 through C_8.

D. BENEFIT OF A LINEAR PROGRAMMING APPROACH IN THE AIR FORCE

The primary usefulness of the linear programming approach in the Indonesian Air Force is that it clearly highlights the trade-offs involved in the decision making and suggests many possible solutions rather than just one solution. It allows tradeoff analyses to be made of what maintenance activities to do "in house" and which to "contract out." For example, in the optimal solution of the example, it shows that Air Force should repair 236 units of communication equipment, 205 units of navigation equipment, 148.833 (149) units of airborne radar equipment, and 144 units of guidance equipment "in house", and "contract out" 6.167 (6) units of communication equipment. The technique also allows us to demonstrate the robustness of an optimal solution in problem immediately. For example, if you are considering changing the cost coefficient for repairing a unit of equipment, or you are considering changing the right-hand side value of a constraint, you can quickly determine if it will have an effect on your optimal solution.

Disadvantages of the technique are that the technique is fairly complex. The quality of the answer is dependent on the quality of the formulation and input values of the parameters. Also, there is the danger of the problem being over simplified in order to remain manageable.

The advantages would allow the Air Force to develop its maintenance organization strategically in order to become the center of excellence of the Air Force. On the other hand, the disadvantages should compel the Air Force to pay attention to the formulation of the model. The Air Force will need good data on parameter values to build the model. One possible way to gather such data precisely and quickly is to design a maintenance management data collection system.

1. Maintenance Management System

Maintaining equipment history is an important part of a maintenance system, as is a record of when, the time it took, and the cost to make the repair. The United States Navy has such a system. It is called the Depot Maintenance Data System.

Preventative maintenance implies that we can determine when a system needs service or will need repair and modification. Corrective maintenance, on the other hand, is the result of unpredictable failures. Therefore, we have to define when a system requires planned service or when it is likely to fail. We also need to know how much of our depot resources are consumed for each type of maintenance.

Figure 10 shows the major components of a computer-based maintenance management system. Such a system would provide a decision maker with information and inputs for a linear programming model for strategic planning. The system would provide information for inventory management, ordering repair parts or a PERT chart for planning a major aircraft overhaul. The system would also provide the data from which to generate the input-output coefficients needed for building a linear programming model.

2. Future Development

By implementing linear programming modeling in the avionics maintenance organization, the Air Force could identify the significant sources of cost per year and it would allow the Air Force to identify where it needs to spend more to increase limited capacity. Avionics maintenance is a military organization requiring huge investments in the Air Force (such as facilities or training personnel). More efficient Air Force maintenance planning will contribute not only to improved defense readiness, but also to national development.

For example, perhaps the Air Force's maintenance facilities will be able to repair avionics equipment not only for the Air Force, but also for Indonesian Aircraft Industry (IPTN), private sector organizations, or the air forces of other Asian countries.

E. SUMMARY

This chapter has presented a discussion of a linear programming example such as might be encountered in aircraft maintenance strategic planning. It spelled out the formulation steps and discussed the optimal solution and sensitivity analysis of the solution. It then extended these basic concepts to a discussion of linear programming's advantages and disadvantages and the problems of gathering data to use in a real world model.

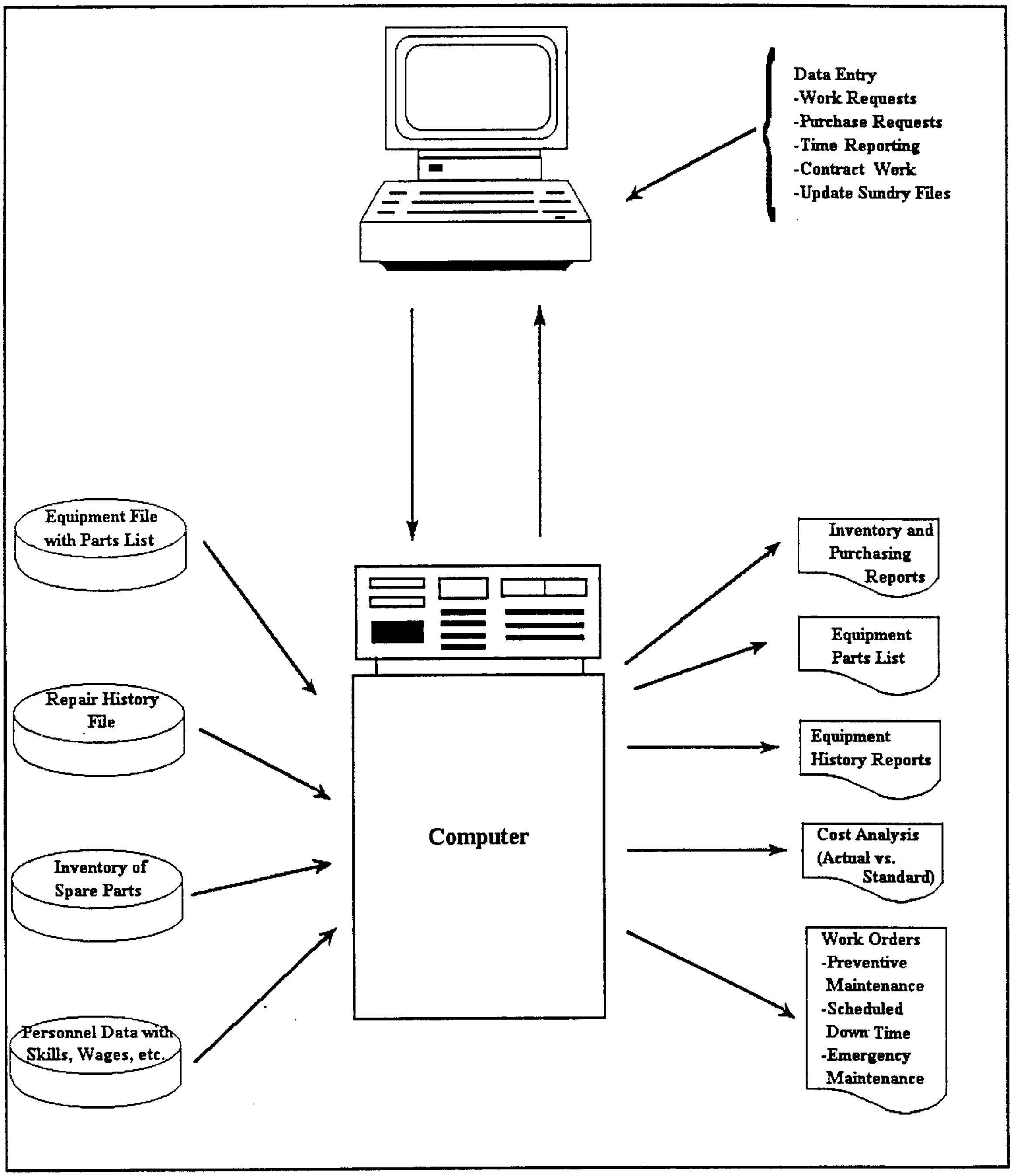

Figure 10. **A Typical Computer-based Maintenance Management System [Ref. 12].**

IV. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY

This thesis has presented the background for a proposed implementation of linear programming in the Indonesian Air Force. The objectives and scope of the thesis were discussed in Chapter I. Chapter II presented the theory needed for subsequent discussion and the application of linear programming to Indonesian Air Force avionics maintenance activities. After determining the benefit desired, the objective function of linear programming can be either maximized or minimized. As we already know in military organizations the linear programming model should optimize readiness or minimize the life cycle costs. Also we presented some examples of real world problems in both civilian and military aviation organizations. Chapter III presented a linear programming example problem in avionics maintenance activities. Chapter III provided the formulation steps and a discussion of the optimal solution and sensitivity analysis of that solution. Chapter III also extended the basic concepts of linear programming to a discussion of linear programming's advantages and disadvantages. Finally, a computer-based management data collection system was suggested to gather the input data required for model formulation.

B. CONCLUSIONS

Although the example considered in this thesis is naive, it does suggest that linear programming might be a useful tool for planning of maintenance activities by the Indonesian Air Force. Linear programming is designed to solve large complex constrained resource problems for the optimal activity levels which will minimize total costs or maximize total profit over a specified time period. The resource constraints and the objective function must be able to be described as linear functions of the unknown activity levels. Linear programming also assumes that the world is deterministic (i.e., any stochastic influences are assumed to be negligible). Fortunately, there are enough real world problems which can be solved in spite of these somewhat limiting assumptions that there are many software papckages available to quickly solve such problems and provide sensitivity analyses of how good the optimal solutions are.

C. RECOMMENDATIONS

Reports of successful linear programming application abound in the literature. Some of these are briefly described at the end of Chapter II and in Appendix A. Others are listed in the Bibliography. Before attempting any application of the linear programming approach these references should be carefully read and understood. In addition, there are undoubtedly some successful applications in the private sector in Indonesia. These need to be sought out and studied.

Modeling help will be needed and can be provided by people who are experts in linear programming and its applications. This author does not happen to be one of them. However, there are faculty members in Operations Research, Mathematics, or Industrial Engineering Departments in Indonesia's major universities, such as Indonesian University (UI), Institute of Technology Bandung (ITB), and Sekolah Teknologi Tinggi Angkatan Laut (STTAL), who can provide assistance and guidance.

A computer-based depot level maintenance management system must be developed to provide the data needed for inputs to linear programming problems. Such systems exist in the United States in both the Air Force and the Navy as well as in the major airlines. These should be studied in detail as a starting point for the development of such a system for the Indonesian Air Force. Again, expert assistance will be needed from universities or private sector corporations which already have such systems.

APPENDIX A. TWO EXAMPLES OF LINEAR PROGRAMMING APPLICATION IN THE AREAS NOT RELATED TO AVIATION

A. CABINET MANUFACTURING

A linear programming model revealed that Cabinet, Inc. can save about $412,000 in raw material cost by purchasing more lower-grade lumber. The company developed a linear programming model of its blank production system. It was structured to minimize the total cost of producing blanks for typical five-day week of operation. The company chooses a five-day planning horizon because it represents the normal cutting cycle of the rough mill; that is, all blank sizes are cut at least once during that period. The constraints include the capacities of the saw mill and dry kilns, the required output of blanks at the manufacturing plant, and the available supply of raw materials. The linear programming problem was:

Minimize the total cost of producing blanks;

subject to:

- Sawmill Capacity
- Drying Capacity
- Weekly Requirements
- Log Supply
- Lumber Supply

Prior to the completion of the model analysis, the company was unable to accurately determine its cost of solid wood raw material. Now the company is able to enter projected procurement conditions and use the results to minimize its cost to produce hardwood blanks. The company feels that the system will give it an advantage in the very competitive cabinet market and allow the company to continue to grow [Ref. 3].

B. CANADA SYSTEMS GROUP

A linear programming model helps the Financial Services Groups division of Canada Systems Group, Incorporated, to cope with the seasonal surge in processing demand for the registered retirement savings program (RRSP, a particular retirement savings plan offered by Canadian Services Groups clients who are fund management companies). During the early part of 1985, the Financial Services Group began to plan its manpower needs for the upcoming surge in transaction processing demand (notifying all parties involved with each fund transaction) of the popular and growing registered retirement savings program. The development of a model and its careful implementation produced, in spite of a 25 percent increase in volume, a savings of over $300,000 over a six-week period as compared to the previous year. [Ref. 24]

APPENDIX B. SOLUTION OF THE PROBLEM

A. SOLUTION OF PRIMAL PROBLEM

This appendix shows the required inputs to the ABQM software and resulting output from solving the linear programming problem of the Air Force's maintenance operation. The outputs include the optimal solution, a sensitivity analysis of it, and the duality analysis of it.

Program: Linear Programming

Problem Title : Avionics Maintenance Operation

***** Input Data *****

Min. Z = 1500x1 + 1250x2 + 2250x3 + 7750x4 + 4750x5 + 4750x6 + 8000x7 + 24250x8

Subject to

C1 .25x1 + .25x2 + .25x3 + .25x4 <= 1540

C2 .25x1 + .25x2 + .50x3 + .50x4 <= 770

C3 5x1 + 6x2 + 7x3 + 8x4 <= 52360

C4 .75x1 + .75x2 + 1.50x3 + 1.50x4 <= 770

C5 1x1 + 1x5 >= 236

C6 1x2 + 1x6 >= 205

C7 1x3 + 1x7 >= 155

C8 1x4 + 1x8 >= 144

***** Program Output *****

Final Optimal Solution At Simplex Tableau : 5

Z =2110458.333

Variable	Value	Reduced Cost
x 1	236.000	0.000
x 2	205.000	0.000
x 3	148.833	0.000

x 4	144.000	0.000
x 5	0.000	375.000
x 6	0.000	625.000
x 7	6.167	0.000
x 8	0.000	10750.000

Constraint	Slack/Surplus	Shadow Price
C 1	1356.542	0.000
C 2	513.333	0.000
C 3	47756.167	0.000
C 4	0.000	3833.333
C 5	0.000	-4375.000
C 6	0.000	-4125.000
C 7	0.000	-8000.000
C 8	0.000	-13500.000

Objective Coefficient Ranges

Variables	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
x 1	-2875.000	1500.000	1875.000	375.000	4375.000
x 2	-2875.000	1250.000	1875.000	625.000	4125.000
x 3	1500.000	2250.000	8000.000	5750.000	750.000
x 4	-5750.000	7750.000	18500.000	10750.000	13500.000
x 5	4375.000	4750.000	No limit	No limit	375.000
x 6	4125.000	4750.000	No limit	No limit	625.000
x 7	2250.000	8000.000	8750.000	750.000	5750.000
x 8	13500.000	24250.000	No limit	No limit	10750.000

Right Hand Side Ranges

Constraints	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
C 1	183.458	1540.000	No limit	No limit	1356.542
C 2	256.667	770.000	No limit	No limit	513.333
C 3	4603.833	52360.000	No limit	No limit	47756.167
C 4	546.750	770.000	779.250	9.250	223.250
C 5	223.667	236.000	533.667	297.667	12.333
C 6	192.667	205.000	502.667	297.667	12.333
C 7	148.833	155.000	No limit	No limit	6.167
C 8	137.833	144.000	292.833	148.833	6.167

***** End of Output *****

APPENDIX C. SHADOW PRICE TEST (RIGHT-HAND SIDE CHANGE)

A. CONSTRAINT C_5 CHANGE

This section shows the effect of reducing the requirement by one unit for the annual repair of communication equipment (right-hand change) and the resulting improvement in the objective function (reduced total cost). The new optimal solution has only X_7 reduced from 6.167 to 5.667 (i.e., one-half less piece of airborne radar equipment is repaired through a contract), and X_3 increased from 148.833 to 149.333 (i.e., one-half more piece of airborne radar equipment is repaired in house). The objective function value decreases by $4,375 (from $2,110,458 to $2,106,083). The decrease is exactly equal to the constraint's shadow price.

Maintenance Operation

Program: Linear Programming

Problem Title : Maintenance Operation

***** Input Data *****

Min. Z = 1500x1 + 1250x2 + 2250x3 + 7750x4 + 4750x5 + 4750x6 + 8000x7
+ 24250x8

Subject to

C1 .25x1 + .25x2 + .25x3 + .25x4 <= 1540

C2 .25x1 + .25x2 + .50x3 + .50x4 <= 770

C3 5x1 + 6x2 + 7x3 + 8x4 <= 52360

C4 .75x1 + .75x2 + 1.50x3 + 1.50x4 <= 770

C5 1x1 + 1x5 >= 235

C6 1x2 + 1x6 >= 205

C7 1x3 + 1x7 >= 155

C8 1x4 + 1x8 >= 144

***** Program Output *****

Final Optimal Solution At Simplex Tableau : 5

Z =2106083.333

Variable	Value	Reduced Cost
x 1	235.000	0.000
x 2	205.000	0.000
x 3	149.333	0.000
x 4	144.000	0.000
x 5	0.000	375.000
x 6	0.000	625.000
x 7	5.667	0.000
x 8	0.000	10750.000

Constraint	Slack/Surplus	Shadow Price
C 1	1356.667	0.000
C 2	513.333	0.000
C 3	47757.667	0.000
C 4	0.000	3833.333
C 5	0.000	-4375.000
C 6	0.000	-4125.000
C 7	0.000	-8000.000
C 8	0.000	-13500.000

Objective Coefficient Ranges

Variables	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
x 1	-2875.000	1500.000	1875.000	375.000	4375.000
x 2	-2875.000	1250.000	1875.000	625.000	4125.000

x 3	1500.000	2250.000	8000.000	5750.000	750.000
x 4	-5750.000	7750.000	18500.000	10750.000	13500.000
x 5	4375.000	4750.000	No limit	No limit	375.000
x 6	4125.000	4750.000	No limit	No limit	625.000
x 7	2250.000	8000.000	8750.000	750.000	5750.000
x 8	13500.000	24250.000	No limit	No limit	10750.000

Right Hand Side Ranges

Constraints	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
C 1	183.333	1540.000	No limit	No limit	1356.667
C 2	256.667	770.000	No limit	No limit	513.333
C 3	4602.333	52360.000	No limit	No limit	47757.667
C 4	546.000	770.000	778.500	8.500	224.000
C 5	223.667	235.000	533.667	298.667	11.333
C 6	193.667	205.000	503.667	298.667	11.333
C 7	149.333	155.000	No limit	No limit	5.667
C 8	138.333	144.000	293.333	149.333	5.667

***** End of Output *****

B. CONSTRAINT C_6 CHANGES

This section shows the effect of reducing the requirement by one unit for the annual repair of navigation equipment (right-hand change) and the resulting improvement in the objective function (reduced total cost). The new optimal solution have X_7 reduced from 6.167 to 5.667 (i.e., one-half less piece of airborne radar equipment is repaired through a contract), and X_3 increased from 148.333 to 149.333 (i.e., one more piece of airborne radar equipment is repaired in house). The objective function value decreases by $4,125 (from $2,110,458.333 to $2,106,333.333). The decrease is exactly equal to the constraint's shadow price.

Program: Linear Programming

Problem Title : Indonesian Air Force's Maintenance Operation

***** Input Data *****

Min. Z = 1500x1 + 1250x2 + 2250x3 + 7750x4 + 4750x5 + 4750x6 + 8000x7 + 24250x8

Subject to

C1 .25x1 + .25x2 + .25x3 + .25x4 <= 1540

C2 .25x1 + .25x2 + .50x3 + .50x4 <= 770

C3 5x1 + 6x2 + 7x3 + 8x4 <= 52360

C4 .75x1 + .75x2 + 1.50x3 + 1.50x4 <= 770

C5 1x1 + 1x5 >= 236

C6 1x2 + 1x6 >= 204

C7 1x3 + 1x7 >= 155

C8 1x4 + 1x8 >= 144

***** Program Output *****

Final Optimal Solution At Simplex Tableau : 5

Z =2106333.333

Variable	Value	Reduced Cost
x 1	236.000	0.000
x 2	204.000	0.000
x 3	149.333	0.000
x 4	144.000	0.000
x 5	0.000	375.000
x 6	0.000	625.000
x 7	5.667	0.000
x 8	0.000	10750.000

Constraint	Slack/Surplus	Shadow Price
C 1	1356.667	0.000
C 2	513.333	0.000
C 3	47758.667	0.000
C 4	0.000	3833.333
C 5	0.000	-4375.000
C 6	0.000	-4125.000
C 7	0.000	-8000.000
C 8	0.000	-13500.000

Objective Coefficient Ranges

Variables	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
x 1	-2875.000	1500.000	1875.000	375.000	4375.000
x 2	-2875.000	1250.000	1875.000	625.000	4125.000
x 3	1500.000	2250.000	8000.000	5750.000	750.000
x 4	-5750.000	7750.000	18500.000	10750.000	13500.000
x 5	4375.000	4750.000	No limit	No limit	375.000
x 6	4125.000	4750.000	No limit	No limit	625.000
x 7	2250.000	8000.000	8750.000	750.000	5750.000
x 8	13500.000	24250.000	No limit	No limit	10750.000

Right Hand Side Ranges

Constraints	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
C 1	183.333	1540.000	No limit	No limit	1356.667
C 2	256.667	770.000	No limit	No limit	513.333
C 3	4601.333	52360.000	No limit	No limit	47758.667
C 4	546.000	770.000	778.500	8.500	224.000
C 5	224.667	236.000	534.667	298.667	11.333
C 6	192.667	204.000	502.667	298.667	11.333
C 7	149.333	155.000	No limit	No limit	5.667
C 8	138.333	144.000	293.333	149.333	5.667

***** End of Output *****

C. CONSTRAINT C_7 CHANGES

This section shows the effect of reducing the requirement by one unit for the annual repair of airborne radar equipment (right-hand change) and the resulting improvement in the objective function (reduced total cost). The new optimal solution have X_7 reduced from 6.167 to 5.167 (i.e., one less piece of airborne radar equipment is repaired through a contract). The objective function value decreases by $8,000 (from $2,110,458.333 to $2,102,458.333). The decrease is exactly equal to the constraint's shadow price.

Program: Linear Programming

Problem Title : Indonesian Air Force's Maintenance Operation

***** Input Data *****

Min. Z = 1500x1 + 1250x2 + 2250x3 + 7750x4 + 4750x5 + 4750x6 + 8000x7 + 24250x8

Subject to

C1 .25x1 + .25x2 + .25x3 + .25x4 <= 1540

C2 .25x1 + .25x2 + .50x3 + .50x4 <= 770

C3 5x1 + 6x2 + 7x3 + 8x4 <= 52360

C4 .75x1 + .75x2 + 1.50x3 + 1.50x4 <= 770

C5 1x1 + 1x5 >= 236

C6 1x2 + 1x6 >= 205

C7 1x3 + 1x7 >= 154

C8 1x4 + 1x8 >= 144

***** Program Output *****

Final Optimal Solution At Simplex Tableau : 5

Z =2102458.333

Variable	Value	Reduced Cost
x 1	236.000	0.000
x 2	205.000	0.000
x 3	148.833	0.000
x 4	144.000	0.000
x 5	0.000	375.000
x 6	0.000	625.000
x 7	5.167	0.000
x 8	0.000	10750.000

Constraint	Slack/Surplus	Shadow Price
C 1	1356.542	0.000
C 2	513.333	0.000
C 3	47756.167	0.000
C 4	0.000	3833.333
C 5	0.000	-4375.000
C 6	0.000	-4125.000
C 7	0.000	-8000.000

C 8	0.000	-13500.000

Objective Coefficient Ranges

Variables	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
x 1	-2875.000	1500.000	1875.000	375.000	4375.000
x 2	-2875.000	1250.000	1875.000	625.000	4125.000
x 3	1500.000	2250.000	8000.000	5750.000	750.000
x 4	-5750.000	7750.000	18500.000	10750.000	13500.000
x 5	4375.000	4750.000	No limit	No limit	375.000
x 6	4125.000	4750.000	No limit	No limit	625.000
x 7	2250.000	8000.000	8750.000	750.000	5750.000
x 8	13500.000	24250.000	No limit	No limit	10750.000

Right Hand Side Ranges

Constraints	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
C 1	183.458	1540.000	No limit	No limit	1356.542
C 2	256.667	770.000	No limit	No limit	513.333
C 3	4603.833	52360.000	No limit	No limit	47756.167
C 4	546.750	770.000	777.750	7.750	223.250
C 5	225.667	236.000	533.667	297.667	10.333
C 6	194.667	205.000	502.667	297.667	10.333
C 7	148.833	154.000	No limit	No limit	5.167
C 8	138.833	144.000	292.833	148.833	5.167

***** End of Output *****

D. CONSTRAINT C_8 CHANGES

This section shows the effect of reducing the requirement by one unit for the annual repair of air guidance systems equipment (right-hand change) and the resulting improvement in the objective function (reduced total cost). The new optimal solution have X_7 reduced from 6.167 to 5.167 (i.e., one less piece of air guidance system equipment is repaired through a contract), and X_3 increased from 148.833 to 149.833 (i.e., one more piece of air airborne radar system equipment are repaired in house). The objective function value decreases by $13,500 (from $2,110,458.333 to $2,096,958.333). The decrease is exactly equal to the constraint's shadow price.

Program: Linear Programming

Problem Title : Maintenance Operation

***** Input Data *****

Min. Z = 1500x1 + 1250x2 + 2250x3 + 7750x4 + 4750x5 + 4750x6 + 8000x7
+ 24250x8

Subject to

C1 .25x1 + .25x2 + .25x3 + .25x4 <= 1540

C2 .25x1 + .25x2 + .50x3 + .50x4 <= 770

C3 5x1 + 6x2 + 7x3 + 8x4 <= 52360

C4 .75x1 + .75x2 + 1.50x3 + 1.50x4 <= 770

C5 1x1 + 1x5 >= 236

C6 1x2 + 1x6 >= 205

C7 1x3 + 1x7 >= 155

C8 1x4 + 1x8 >= 143

***** Program Output *****

Final Optimal Solution At Simplex Tableau : 5

Z =2096958.333

Variable	Value	Reduced Cost
x 1	236.000	0.000
x 2	205.000	0.000
x 3	149.833	0.000
x 4	143.000	0.000
x 5	0.000	375.000
x 6	0.000	625.000
x 7	5.167	0.000
x 8	0.000	10750.000

Constraint	Slack/Surplus	Shadow Price
C 1	1356.542	0.000
C 2	513.333	0.000
C 3	47757.167	0.000
C 4	0.000	3833.333
C 5	0.000	-4375.000
C 6	0.000	-4125.000
C 7	0.000	-8000.000
C 8	0.000	-13500.000

Objective Coefficient Ranges

Variables	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
x 1	-2875.000	1500.000	1875.000	375.000	4375.000
x 2	-2875.000	1250.000	1875.000	625.000	4125.000

x 3	1500.000	2250.000	8000.000	5750.000	750.000
x 4	-5750.000	7750.000	18500.000	10750.000	13500.000
x 5	4375.000	4750.000	No limit	No limit	375.000
x 6	4125.000	4750.000	No limit	No limit	625.000
x 7	2250.000	8000.000	8750.000	750.000	5750.000
x 8	13500.000	24250.000	No limit	No limit	10750.000

Right Hand Side Ranges

Constraints	Lower Limit	Current Values	Upper Limit	Allowable Increase	Allowable Decrease
C 1	183.458	1540.000	No limit	No limit	1356.542
C 2	256.667	770.000	No limit	No limit	513.333
C 3	4602.833	52360.000	No limit	No limit	47757.167
C 4	545.250	770.000	777.750	7.750	224.750
C 5	225.667	236.000	535.667	299.667	10.333
C 6	194.667	205.000	504.667	299.667	10.333
C 7	149.833	155.000	No limit	No limit	5.167
C 8	137.833	143.000	292.833	149.833	5.167

***** End of Output *****

APPENDIX D. CALCULATION OF CAPACITY OF DEPOT LEVEL AVIONICS MAINTENANCE ACTIVITIES

This appendix provides estimates of the capacity in the four phases of depot level avionics maintenance performed by the Indonesian Air Force based on my experience from 1985 to 1991. In this case we will calculate the capacity of each repair work station based on an effective working-day.

- Monday - Thursday: 6 hours/day. x 4 days = 24 hours
- Friday 3.5 hours. x 1 day = 3.5 hours
- Saturday 3.5 hours. x 1 day = 3.5 hours

Total/hours/week = 31 hours. So, the average capacity for each test bench and work station is 31 hours/week. If there are 52 weeks during year, then the hours/year = 31 x 52 = 1612 hours. However, there are 14 holidays during a year then the total holidays-hours during a year = 31 hours/week x 2.33 (14 days = 2.33 weeks) = 72.33 = 72 hours (rounding down). Therefore, the total available hours/year for each work station becomes = 1612 hours - 72 hours = 1540 hours. Table 6 is a detailed capacity analysis of the work stations involved in avionics maintenance.

No DESCRIPTION TYPE	QUANTITY OF WORK STATIONS	CAPACITY HRS/YRS
1. Communication Equipment		
a. HF (High Frequency)		
618T-1/2	1	1540
628T-2	1	1540
718U-5	2	3080
b. VHF (Very High Frequency)		
618M-3	1	1540
VHF20()	1	1540
610F-1	1	1540
VHF253	1	1540
ARC-131	1	1540
c. UHF (Ultra High Frequency)		
ARC159	1	1540
ARC182	1	1540
d. Intercom		
AIC-18	1	1540
AIC-10	1	1540
Total		20,020

No DESCRIPTION TYPE	QUANTITY OF WORK STATIONS	CAPACITY HRS/YRS
2. Navigation Equipment		
a. VHF Navigation		
51RV-()	1	1540
VIR-30()	1	1540
b. ADF (Automatic Direction Finder)		
DF-206	2	3080
c. Altimeter		
AL-101	1	1540
d. MarkerBeacon		
51Z-4	1	1540
e. UHF/VHF DF (Direction Finder)		
ARA-50	1	1540
DF-301	1	1540
Total		12,320
3. Airborne Radar Equipment		
a. Weather Radar		
RDR/400	2	3080
RDR-IF/E/FB	2	3080
b. Weather/Mapping Radar		
AVQ30	1	1540
APQ59	1	1540
c. SLAMMR (Side Looking Airborne Multi-Mission Radar)		
	1	1540
Total		10,780

No DESCRIPTION TYPE	QUANTITY OF WORK STATIONS	CAPACITY HRS/YRS
4. Guidance System Equipment		
a. INS (Inertial Navigation System)		
LTN-72	1	1540
b. ONS (Omega Navigation System)		
CMA-740/771	2	3080
c. Compass System		
C-12	1	1540
d. Autopilot		
FCS-105	1	1540
e. GPWS		
Mark-II	1	1540
Total		9,240

Table 6. Detailed Capacity of Work Stations Used for Avionics Depot Level Repair.

So that, the total capacity (measured in hours per year) of the depot level avionics work stations per year is:

1. Communication	20,020 hours/year
2. Navigation	12,320 hours/year
3. Airborne Radar	10,780 hours/year
4. Guidance System	9,240 hours/year

Table 7. Total Capacity of Repair Work Stations Per Year.

Therefore the total capacity for the repairing process is about 52,360 hours (20,020 + 12,320 + 10,780 + 9,240). It is important to note that it may actually be less because the repair technicians are required for other activities (outside duty). Next, we need to consider the capacity of the activities preceding and following repair. These are those in Phases I, II, and IV. Assuming a single work station for each, we get:

- Receiving, visual check, work order preparation are also done 31 hours/week, and 31 x 52 (weeks per year) minus the holidays leaves 1540 hours (in the real world it may be less).
- The pre-inspection check is only done half of the time during each week. Thus, the capacity of pre-inspection/year = 1/2 x 1540 = 770 hours.
- Similarly, final inspection is currently done only half of the time during each week. Thus, the capacity of the final inspection station is 770 hours/year.

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